A Pauli-Sylvester Approach
for Calibration and Validation of ALOS2 Quad-pol Data

by

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Outline

1. Introduction: Polarimetric Calibration Model and FR challenge at L-band

2. New Calibration Algorithm: Pauli-Sylvester
   - Generate similarity transformation from ‘mirror’ scattering data (Sylvester equation)
   - Expand using the Pauli matrices to generate a ‘sparse’ matrix solution
   - Solve using a network of natural point targets..no clutter averaging..
   - get some invariance to Faraday Rotation..can apply it anywhere...

3. Sample L-Band ALOS2 results for our Victoria calibration site

4. Generalise technique to help validate scenes without ‘mirror’ targets

5. Results for all 5 quadpol beam modes of ALOS-PALSAR

6. Conclusions
L-band Distortion Model

L-Band model = TX-Faraday-scatter-Faraday-RX

\[
[S]_{bs} = [R][F][S][F][T] + [N] = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \begin{bmatrix} \cos \theta_f & -\sin \theta_f \\ \sin \theta_f & \cos \theta_f \end{bmatrix} \begin{bmatrix} hh & hv \\ vh & vv \end{bmatrix} \begin{bmatrix} \cos \theta_f & -\sin \theta_f \\ \sin \theta_f & \cos \theta_f \end{bmatrix} \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} + [N]
\]

Simplified form used for algorithm development..

\[
[S]_{bs} = [R][S][T] + [N] = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \begin{bmatrix} hh & hv \\ vh & vv \end{bmatrix} \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} + [N]
\]

Small coupling hypothesis \( r_{ij}, t_{ij} \ll r_{ii}, t_{ii} \) …but uncompensated FR can masquerade as false cross-talk.

Reciprocity \( hv = vh \)
The Classic Approach: Dense matrices

Vectorise [S] in lexicographic form

\[
\begin{bmatrix}
hh \\
hv \\
vh \\
vv
\end{bmatrix} = \frac{1}{Y} \begin{bmatrix}
k^2 & 0 & 0 & 0 \\
0 & k & 0 & 0 \\
0 & 0 & k & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
[Z]^{-1} \begin{bmatrix}
hh \\
hv \\
vh \\
vv
\end{bmatrix}_{\text{obs}}
\end{bmatrix}
\]

\[Y = r_{22}t_{22}, k = \frac{r_{11}}{r_{22}}\]

Obtained from a trihedral

\([Z]^{-1}\) is a dense matrix with R and T dependence

\[
[Z]^{-1} = \frac{1}{a} \begin{bmatrix}
1 & -v & -w & vw \\
-az & a & azw & -aw \\
-u & uv & 1 & -v \\
azu & -au & -az & a
\end{bmatrix}
\]

\[u = \frac{r_{21}}{r_{11}}, v = \frac{t_{21}}{t_{22}}, w = \frac{r_{12}}{r_{22}}, z = \frac{t_{12}}{t_{11}}, a = \frac{r_{22}t_{11}}{r_{11}t_{22}}\]

Obtained from clutter assuming e.g. \(<hh.hv^*> = 0\) etc.
A New Approach:
Sylvester equation in Pauli form*

Vectorise $[S]$ in **Pauli** form

\[
\begin{bmatrix}
p_1 \\
p_2 \\
p_3 \\
p_4
\end{bmatrix} = [Z]^{-1} \cdot \begin{bmatrix}
p_1 \\
p_2 \\
p_3 \\
p_4_{obs}
\end{bmatrix} \quad [S]^{-1}_{obs} = [T]^{-1}[R]^{-1}
\]

Obtained from a trihedral

$[Z]^{-1}$ is then a sparse matrix with only T (or R) dependence

\[
[Z]^{-1} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & u & 0 \\
0 & v & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Obtained from reciprocity theorem

\[
u = \frac{t_{12}t_{11} - t_{21}t_{22}}{t_{11}t_{22}} \\
v = \frac{2(t_{21}t_{11} - t_{12}t_{22})}{t_{11}^2 + t_{22}^2}
\]

N.B. only one of $u$ or $v$ is required to fully calibrate

\[|u|, |v| << 1\]

Using Full Information from Trihedrals: The Sylvester Equation

Measurement of a mirror (CR) yields a 2x2 complex scattering matrix:

\[
[S] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow [S]_{CR}^{obs} = [R][T]
\]

We can use this to remove completely the receiver (or transmitter cal errors)...

\[
[S]_{obs} = [R][S][T] \Rightarrow [S]_{CR}^{obs} = [T]^{-1}[R]^{-1}[R][S][T] = [T]^{-1}[S][T]
\]

Notes:

1) If we apply the inverse CR S-matrix to each SLC pixel we remove receiver/transmit distortion.
2) Then we must estimate T or R from \(S_{obs}\)…this is a classical problem…the Sylvester equation.
3) Importantly now all system errors follow a similarity transformation of \(S\) …so use the Pauli basis.
4) To solve need CR plus N other pixels*…

* no need to know their \([S]\) matrices, just need to be non-trihedral
Solving the Sylvester Equations: Use the Reciprocity Theorem

\[ k = [A]^{-1} k_{\text{obs}} = \begin{bmatrix} k_0 \\ k_1 \\ k_2 \\ 0 \end{bmatrix}, \quad [A]^{-1} = \frac{1}{m(C_1 - x_1 x_2)} \begin{bmatrix} C_1 - x_1 x_2 & 0 & 0 & 0 \\ 0 & C_1 & -x_1 & 0 \\ 0 & -x_2 & 1 & 0 \\ 0 & a_{42} & a_{43} & C_1 - x_1 x_2 \end{bmatrix} \]

Reciprocity theorem..for all pixels in SLC data

\[ k_{\text{obs}} = \begin{bmatrix} z_0 \\ z_1 \\ z_2 \\ z_3 \end{bmatrix} \Rightarrow \frac{a_{42}}{m(C_1 - x_1 x_2)} z_1 + \frac{a_{43}}{m(C_1 - x_1 x_2)} z_2 + \frac{1}{m} z_3 = 0 = k_3 \]

For N pixels...

\[ \frac{1}{C_1 - x_1 x_2} \begin{bmatrix} z_1^1 & z_2^1 \\ \vdots & \vdots \\ z_1^N & z_2^N \end{bmatrix} \begin{bmatrix} a_{42} \\ a_{43} \end{bmatrix} = \begin{bmatrix} -z_3^1 \\ -z_3^2 \\ \vdots \\ -z_3^N \end{bmatrix} \Rightarrow \frac{1}{C_1 - x_1 x_2} \begin{bmatrix} a_{42} \\ a_{43} \end{bmatrix} = Z^+ b \]
Reciprocity Theorem 2 parameters:
Mean X-talk and complex Copolar Channel Imbalance

We can use N pixels (N>=2) to solve the following set of linear equations

\[
\frac{1}{C_1 - x_1 x_2} \begin{bmatrix} a_{42} \\ a_{43} \end{bmatrix} = Z^* b
\]

We use N brightest non-trihedrals along each azimuth line. This helps us avoid SNR problems. The two parameters \( a_{42}, a_{43} \) are related to cross-talk and channel imbalance as follows

\[
\frac{a_{42}}{C_1 - x_1 x_2} = \frac{x_2 C_2 - C_1 x_3}{C_1 - x_1 x_2} = \frac{C_2}{C_1} x_2 - x_3
\]
\[
\frac{a_{43}}{C_1 - x_1 x_2} = \frac{x_1 x_3 - C_2}{C_1 - x_1 x_2} = -\frac{C_2}{C_1}
\]

\( a_{42} = \text{Average cross-talk} \)
\( a_{43} = \text{Channel Imbalance} \)

from \( a_{43} \) we can directly estimate the complex channel imbalance as

\[
-\frac{C_2}{C_1} = \varepsilon = \frac{1 - c^2}{1 + c^2} \Rightarrow c = \pm \sqrt{\frac{1 - \varepsilon}{1 + \varepsilon}} = \pm \frac{t_{22}}{t_{11}} \text{ or } \frac{r_{22}}{r_{11}}
\]
**GVWD CORNER REFLECTOR LOCATIONS**

- **Northeast Shore of Sooke Lake Corner Reflector Location:**
  - Latitude: 48.571481°
  - Longitude: -123.672023°
  - Grid Northing: 5380039.13m
  - Grid Easting: 450426.39m
  - Elevation Height: 184m

- **23 S Quarry (North) Corner Reflector Location:**
  - Latitude: 48.530980°
  - Longitude: -123.708336°
  - Grid Northing: 5375561.41m
  - Grid Easting: 447705.96m
  - Elevation Height: 216m
Sample L-band Results: Victoria 4/9/14 FP6-4 beam

Red = T, Blue = R matrix…even with 1° of FR present!

We want to check other Quadpol Beams..

but don’t have CR in scene for all of them…

What to do?

Low xtalk ….good amplitude balance…phase offset in R?
1\textsuperscript{st} step: Validation of phase anomaly using a different algorithm

Using a variation of Ainsworth Algorithm*, $R$ and $T$ are

$$R = \begin{bmatrix}
(-0.1517 \text{ dB}, -21.6014^\circ) & (-27.3095 \text{ dB}, +157.3579^\circ) \\
(-29.6201 \text{ dB}, -17.6627^\circ) & (0 \text{ dB}, 0^\circ)
\end{bmatrix}$$

$$T = \begin{bmatrix}
(-0.3573 \text{ dB}, +2.7126^\circ) & (-57.3115 \text{ dB}, -73.2207^\circ) \\
(-48.7201 \text{ dB}, +28.2750^\circ) & (0 \text{ dB}, 0^\circ)
\end{bmatrix}$$

[1] Orientation angle preserving a posteriori polarimetric SAR calibration, Ainsworth, etc., 2006, TGRS
[3] Improvement of Polarimetric SAR Calibration Based on the Ainsworth Algorithm for Chinese Airborne PolSAR Data, H. Zhang, etc., 2013, GRSL
What to do if there is no Trihedral?
The Sylvester story

- Full Method: CR + Dihedral, [R] and [T] and Faraday invariant

- CR + Reciprocity: [R] and [T] low cross talk approximation, Faraday invariant
  
  No CR?...use Reciprocity only

- Assume data is pre-calibrated and has low cross-talk, but channel imbalance.
  
  Classical solution in this case (Sarabandi 1990’s)... \[ \frac{hv - vh}{hv + vh} \approx \frac{t_{22} - r_{22}}{t_{22} + r_{22}} = -a_{43} \]

  But for P and L bands Faraday rotation causes error in this approach...
  
  (hv+vh remains invariant to FR... but hv-vh has \( \sin(2\theta) \) (hh+vv) \( \approx a_{42} \) error)

  ..so can use Sylvester \( a_{42} \) estimate as a check of accuracy of r/t estimate..

  Avoid high FR (night time) and ‘natural’ trihedrals... combo gives best results
Pauli-Sylvester Validation Algorithm

We want to know $t_{22}$ and $r_{22}$ as complex unknowns… what we get is $t_{eff}$

$$S_{obs} = \begin{bmatrix} 1 & 0 \\ 0 & r_{22} \end{bmatrix} \begin{bmatrix} S_{HH} & S_{HV} \\ S_{HV} & S_{VV} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & t_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & t_{eff} \end{bmatrix} \begin{bmatrix} S_{HH} & S_{HV} \\ S_{HV} & S_{VV} \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$= [T]^{-1}[S][T] \Rightarrow [R][T] = [I]$$

This implies that we can only estimate the complex ratio..

$$t_{eff} = \sqrt{\frac{t_{22}}{r_{22}}}$$

So $t_{eff}$ should be $= 1$ for well calibrated data… a validation procedure…

If not $= 1$ then can approximately calibrate

Assume $[R]$ is in error, $[T]$ ok (based on our CR example)... so $t_{22} = 1$  $r_{22} = \frac{1}{t_{eff}^2}$
Fort McMurray, ALOS-2, FP6-6, 4/4/2015... 1 year before the fires
Sylvester Validation for Fort McMurray FP6-6 data

Phase anomaly again..
...in a different mode
Fort McMurray Calibration check:
High HHVV Coherence surface scatter

Coherence Unit Circle...
Note phase error (should be ≈ 0°)
Fort McMurray Calibration check:
High HHVV Coherence surface scatter

After correction using Pauli-Sylvester approach

\[ t_{22} = 1 \]

\[ r_{22} = \frac{1}{t_{eff}^2} \]
Summary: High Coherence surface scattering in all 5 polarimetric beams of ALOS-2

Vancouver, Canada, FP-3, 12/3/2016
Cross-pol isolation in all modes is good (-40dB) but phase calibration is a problem for some modes.

<table>
<thead>
<tr>
<th>Stripmap Modes</th>
<th>FBQ</th>
<th>FBC</th>
<th>HBQ</th>
<th>HBC</th>
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<tr>
<td>Polarization</td>
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<td>Res/Swath</td>
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<tr>
<td>Beam (AOI)</td>
<td>Quad 10m/70km</td>
<td>Compact 10m/70km</td>
<td>Quad 6m/50km</td>
<td>Compact 6m/50km</td>
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<td></td>
<td>FP10 (21.5)</td>
<td>FP10 (21.5)</td>
<td>FP6-1 (17.8)</td>
<td>FP6-1 (17.8)</td>
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<tr>
<td>Beam (AOI)</td>
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<td>FP6-2 (21.5)</td>
<td>FP6-2 (21.5)</td>
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<td>Beam (AOI)</td>
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<td>FP6-3 (25)</td>
<td>FP6-3 (25)</td>
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<tr>
<td>Beam (AOI)</td>
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<td>FP6-4 (28)</td>
<td>FP6-4 (28)</td>
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<td>Beam (AOI)</td>
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<td>FP6-5 (30.4)</td>
<td>FP6-5 (30.4)</td>
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<td>Beam (AOI)</td>
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<td>FP6-6 (32.7)</td>
<td>FP6-6 (32.7)</td>
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<td>Beam (AOI)</td>
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<td>FP6-7 (34.9)</td>
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<tr>
<th>FP6-3(25)</th>
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</tr>
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Conclusions

- New polarimetric calibration algorithm: Pauli-Sylvester
  - Remove [R] or [T] from the equations using a mirror..
  - Sylvester equation...force a similarity transformation..
  - Pauli matrix expansion is a parsimonious way to a sparse matrix solution..

Advantages: fewer equations, simpler inversion, no clutter averaging, FR invariant
Disadvantages: need good SCR around trihedrals, need dihedral for full calibration

- Applied to L-band ALOS2 calval site in Canada; found a phase anomaly in one mode. All data obtained from AUIG2 ALOS User Interface Gateway)

- To check this in other modes (without trihedrals) developed a validation version..

- Applied to all quadpol modes and found phase anomalies in 2 modes
  - FP6-4 and FP6-6...would ask others to validate in these modes...

- Have also applied to other polarimetric sensors, such as PALSAR, Radarsat-2, and Terrasar-X.
Check Pre-calibration in SAR processor:
R and T Matrices in CEOS data header file

'Good' Modes e.g. FP6-3

Phase shift Modes e.g. FP6-4
Faraday Rotation and the Sylvester Equations

FR is benign in the Sylvester approach...it does not masquerade as false x-talk, just a rotation

\[ [T] = [R(\theta)][T_D] = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} t_{11}^d & 0 \\ 0 & t_{22}^d \end{bmatrix} \]

e.g. add some FR to a zero X-talk system...was a good model for ALOS-PALSAR

Distortion model

\[
k_{\text{obs}} = Ak = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & C_1 & 0 \\ 0 & 0 & C_2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -\tan 2\theta & 0 \\ 0 & C_1 \tan 2\theta & C_1 & 0 \\ 0 & C_2 \tan 2\theta & C_2 & 1 \end{bmatrix} k
\]

Solve using reciprocity only

\[
\begin{bmatrix} m & 0 & 0 & 0 \\ 0 & 1 & x_1 & 0 \\ 0 & x_2 & C_1 & 0 \\ 0 & x_3 & C_2 & m \end{bmatrix} \Rightarrow \begin{cases} a_{43} = -\frac{C_2'}{C_1'} = \epsilon \\ \frac{a_{42}}{C_1' - x_1'x_2'} = 0 \end{cases}
\]

zero x-talk measured...i.e. FR is not confused with system cross-talk